

# Statistical Distributions of the Re-Radiated Spectrum from Two Correlated Non-Linear Devices in a Reverberation Chamber

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**Abstract**—The statistics of the re-radiated spectrum from two correlated non-linear devices are investigated in a reverberation chamber. The distribution of the mean-value-normalized statistics is interpreted using a double-Weibull statistical model. Comparisons are made with the re-radiation spectrum of a single non-linear device showing the statistical distributions to be different. Furthermore, experiments indicate the spatial correlation between the two non-linear devices changes the statistical distributions. This work enhances the understanding about the statistical aspects of the re-radiated spectrum from complex digital equipment.

## I. INTRODUCTION

When a digital device is stressed by an external RF interferer, the internal signals will interact with the induced RF energy and cause the re-radiation of an inter-modulation spectrum. The feasibility of utilizing this phenomenon in digital immunity diagnostics was first investigated in [1] in an anechoic chamber. It was discovered that the magnitude of the re-radiated spectrum increases with the level of RF interference until the latter approaches a certain level. Then there is a sudden change of the re-radiated spectrum that can be related to the immunity of the digital device [2].

It is advantageous to use a reverberation chamber for immunity testing because of the high field intensity it can achieve with only a moderate input power [3]. The statistical aspects of the re-radiated spectrum measured in a reverberation chamber have been investigated using a statistical model of cascaded random processes [4]. A more versatile statistical model was then proposed to explain the changing statistical distribution due to the hard nonlinearity of the re-radiating source [5]. These experimental and analytical methods were subsequently applied to research the statistics of the re-radiated inter-modulation spectrum from a modulated diode circuit [6], [7].

The previous research assumes there is only one, dimensionless, re-radiating source interacting with the impinging RF energy, which limits the generality of the deduced conclusions as in reality a complex digital equipment may have multiple elements re-radiating. We

hypothesise that when multiple re-radiating sources exist, the statistical distribution of the re-radiated spectrum will differ from the single source case. Furthermore, we expect that the spatial correlations among the coupling links of the re-radiating sources play a role in determining the distribution.

To validate the hypothesis above, identical diode circuits, modulated by 10 MHz square waves generated by a digital signal generator, have been made. The circuits were connected to monopole antennas which are fixed at different positions on a ground-plane made of copper as shown in Fig.1. The remaining parts of the circuits are screened by metallic boxes to ensure the coupling paths are only through the connectivity with each monopole antenna.

As shown in Fig.1, there are three monopole antennas (numbered as 1, 2 and 3 from left to right) fixed on the test board, each of which can be connected to a diode circuit.

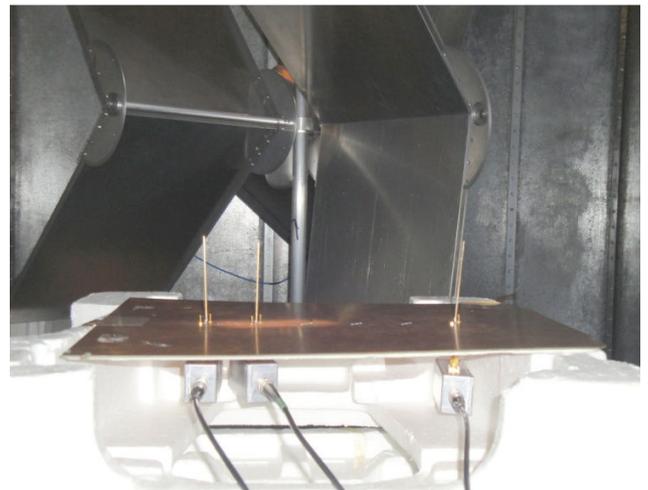


Fig. 1. A photograph of the diode circuits fixed on a test board in the reverberation chamber.

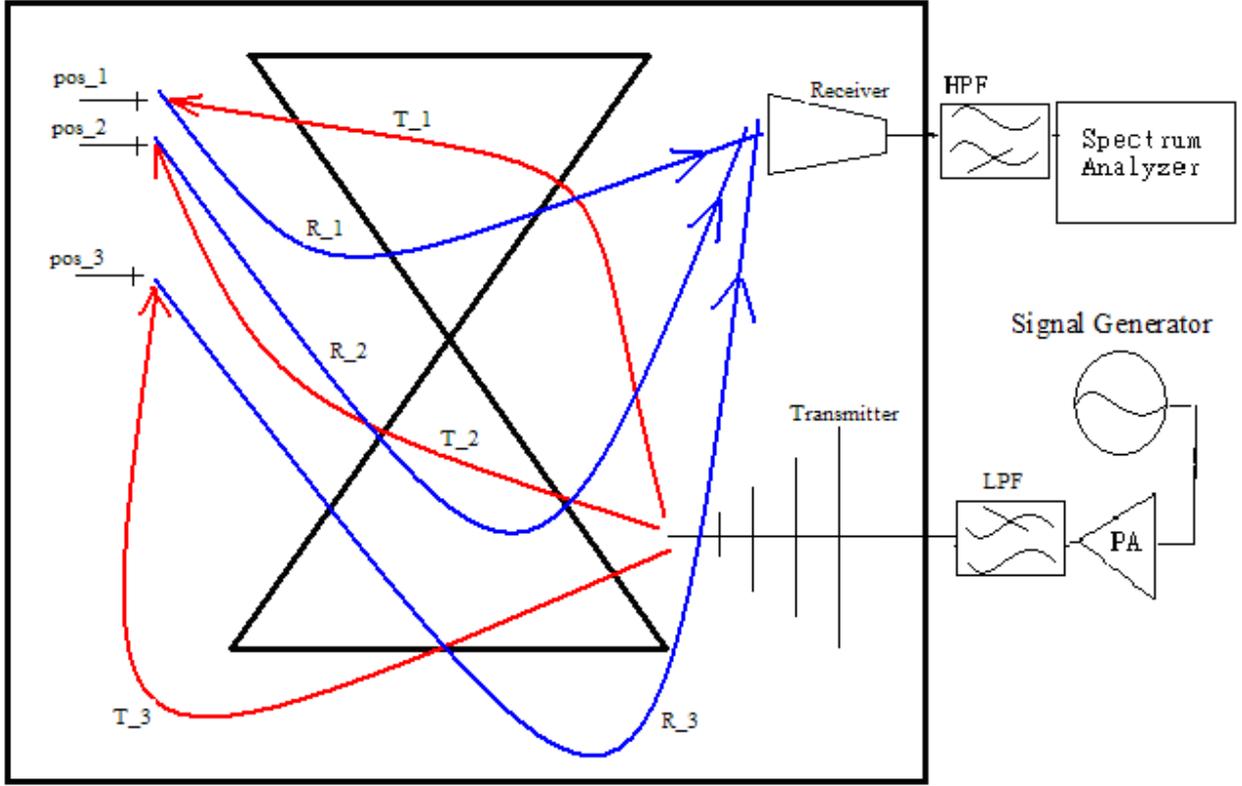


Fig. 2. Experiment setup with three monopole positions for two diode circuits.

The distance between position 1 and 2 is 5 cm; the distance between positions 1 and 3 is 25 cm. By changing the positions of the diode circuits, the statistics of the re-radiated spectrum from multiple correlated nonlinear devices can be investigated experimentally.

## II. EXPERIMENT PROCEDURES AND RESULTS

The experiment procedure for measuring the re-radiated spectrum is the same as that reported in [6] and [7]. The measurement setup is shown in Fig.2. The test board, where monopoles 1, 2 and 3 are fixed, is located within the working volume of the reverberation chamber. The coupling path from the transmitter antenna to the  $i^{th}$  monopole is labelled as  $T_i$  and that from the  $i^{th}$  monopole to the receiver antenna is labelled as  $R_i$ . The statistics of the coupling coefficients of  $T_i$  and  $R_i$  are assumed to follow Rayleigh distributions [3]. In the experiment, the frequency of the RF carrier  $f_c$  is set as 850 MHz and the inter-modulation spectrum at frequency  $f_{IMP} = 2f_c - f_{clk}$  is investigated. When the digital signal frequency,  $f_{clk}$  is 10 MHz, the inter-modulation frequency is 1690 MHz.

The correlation coefficients between the various coupling paths,  $T_i$  and  $T_j$  ( $i, j = 1, 2, 3; i \neq j$ ),  $R_i$  and  $R_j$  ( $i, j = 1, 2, 3; i \neq j$ ), can be calculated from the measured scattering parameters between the monopole antennas. The calculation

results are used to quantify how much the two re-radiating sources are correlated to each other. Supposing the stirrer returns to its original position after one rotation, repeating the measurement at different monopole positions it is equivalent to measuring all the coupling paths at the same stirrer angle. The correlation factors for paths 1 and 2, expressed in matrix form are:

$$\begin{aligned} \text{corr}(T_1, T_2) &= \begin{bmatrix} 1 & 0.807 \\ 0.807 & 1 \end{bmatrix} \\ \text{corr}(R_1, R_2) &= \begin{bmatrix} 1 & 0.487 \\ 0.487 & 1 \end{bmatrix} \end{aligned}$$

The distance between position 1 and 2 is 14% of the wavelength of the impinging RF wave and 28% of the wavelength of the re-radiated field at 1690 MHz. The results above show a strong correlation between position 1 and 2, especially for the incident wave coupling. For positions 1 and 3 the correlation factors are:

$$\begin{aligned} \text{corr}(T_1, T_3) &= \begin{bmatrix} 1 & 0.05 \\ 0.05 & 1 \end{bmatrix} \\ \text{corr}(R_1, R_3) &= \begin{bmatrix} 1 & 0.02 \\ 0.02 & 1 \end{bmatrix} \end{aligned}$$

The distance between position 1 and 3 is 70% of the wavelength of the impinging RF wave and 140% of the wavelength of the re-radiated field at 1690 MHz. The results above show a weak correlation between position 1 and 3, compared with the correlation between position 1 and 2.

A reference measurement of the re-radiated spectrum from one diode circuit, circuit A or B respectively, was then made with the circuit fixed at position 1. It makes little difference if it is fixed at other positions given the field uniformity within the working volume of the reverberation chamber. The input power level to the chamber of the RF energy is set to 25 dBm, and the received spectrum at 1690 MHz for 400 stirrer positions over one rotation is measured by the spectrum analyser as shown in Fig. 2. The statistical distribution of the measurement result was fitted to the double-Weibull distribution developed in our previous work [6], [7]. The probability densities of the mean-normalised-statistics are shown in Fig. 3 and Fig. 4.

Kolmogorov-Smirnov goodness of fit tests show the statistics in Fig.3 and Fig.4 follow the same double-Weibull distribution at 5% significant level with parameters  $k1=1$  and  $k2=2$  [8].

Next circuit A was fixed at position 1 and circuit B at position 2. According to the correlation factors calculated before, the two re-radiating sources are strongly correlated in this configuration. The resulting measured re-radiated spectrum and fitted distribution are shown in Fig.5.

Circuit B was then moved to position 3 and the measurement repeated. This results in two weakly correlated re-radiating sources in the reverberation chamber. The statistical distribution is shown in Fig. 6.

Fig. 7 and Fig.8 show the distributions of a single circuit, A and B, with a chamber input RF power level of 29 dBm. The corresponding results for strongly and weakly correlated re-radiating sources under 29 dBm RF level are shown in Fig.9 and Fig.10 respectively.

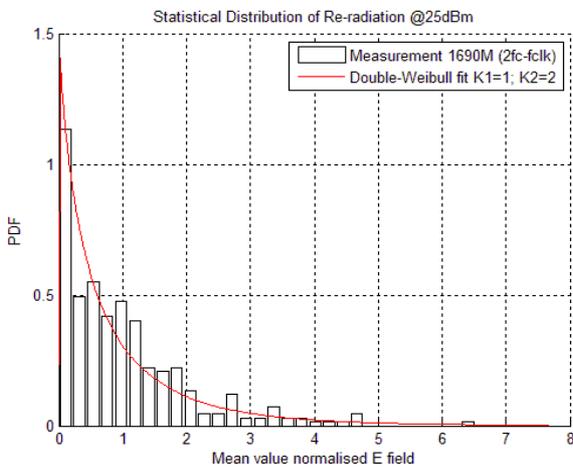


Fig.3 PDF of the inter-modulation spectrum at 1690 MHz with a single re-radiating source (circuit A) under 25dBm RFI.

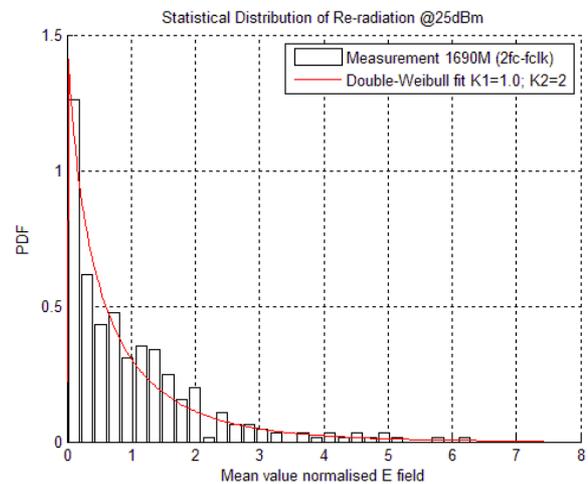


Fig. 5. PDF of the inter-modulation spectrum at 1690 MHz with two strongly correlated re-radiating sources (A and B) under 25 dBm RFI.

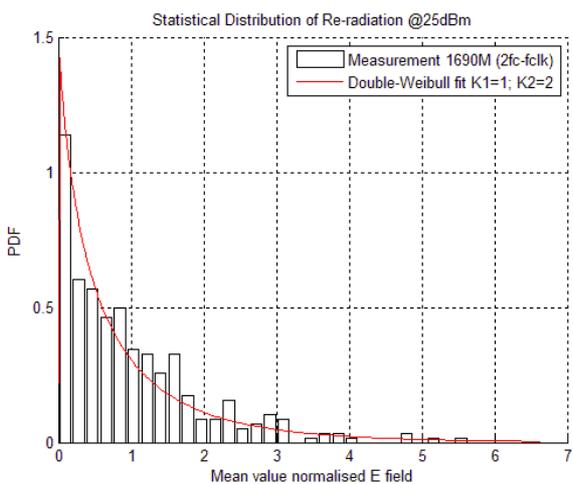


Fig. 4. PDF of the inter-modulation spectrum at 1690 MHz with a single re-radiating source (circuit B) under 25dBm RFI.

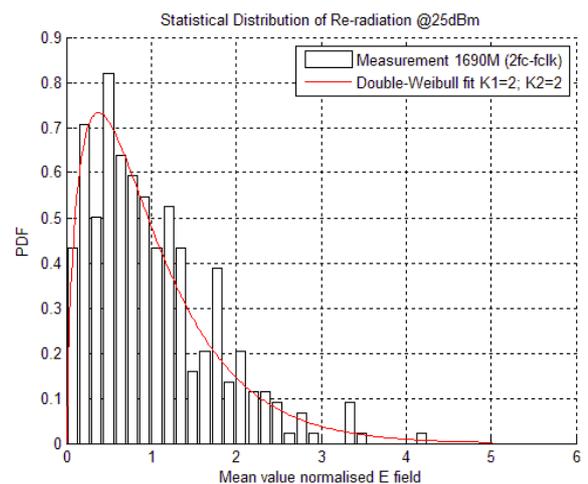


Fig. 6. PDF of the inter-modulation spectrum at 1690 MHz with two weakly correlated re-radiating sources (A and B) under 25 dBm RFI.

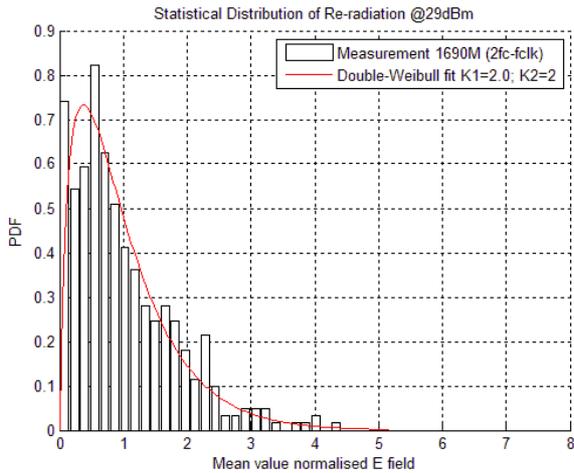


Fig. 7. PDF of the inter-modulation spectrum at 1690 MHz with a single re-radiating source (circuit A) under 29 dBm RFI.

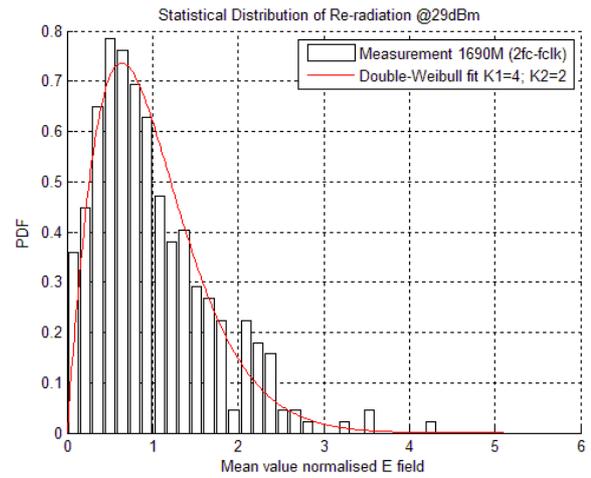


Fig. 10. PDF of the inter-modulation spectrum at 1690 MHz with two weakly correlated re-radiating sources (A and B) under 25 dBm RFI.

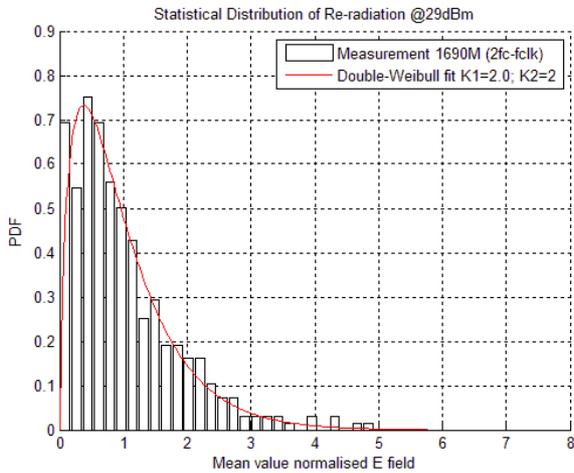


Fig. 8. PDF of the inter-modulation spectrum at 1690 MHz with a single re-radiating source (circuit B) under 29 dBm RFI.

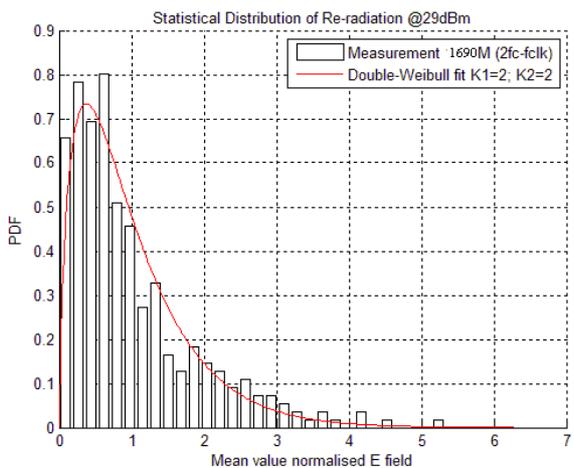


Fig. 9. PDF of the inter-modulation spectrum at 1690 MHz with two strongly correlated re-radiating sources (A and B) under 29 dBm RFI.

### III. DISCUSSION

From the results shown in Fig. 3, Fig. 4 and Fig. 5, it is found that the statistical distribution of the received re-radiated spectrum from two strongly correlated sources is identical to the case of either one re-radiating alone, under the same threat RF power level. When the two re-radiating sources are strongly correlated, the coupling coefficients of the two have strong linear relationship and the combined statistical distribution is similar to the case when there is only one source. This can also be explained as the distance between two sources are too small compared to the wavelength of electromagnetic wave, therefore the field components at the two positions are similar.

However, the statistical distribution of two weakly correlated sources is significantly different, as shown in Fig. 6. The double-Weibull distributions fitted to the results are found to have significantly different shape parameters, manifesting the difference in behaviour. Given that the non-linearities of the circuits are the same, with the same the input RF power level, the only difference is caused by the spatial locations of the two non-linear devices re-radiating in the reverberation chamber.

The results shown in Fig. 7 to Fig. 10 reveal the effects of changing the character of the nonlinearities on the statistical distributions caused by increasing the RF power level. By comparing the results from experiments with the same layout of sources, but with a higher chamber input RF power level, the statistical distribution is found to be dependent on the level of the input RF energy as well.

### IV. CONCLUSIONS

The statistics of the re-radiated spectrum from two correlated non-linear devices have been investigated experimentally. The results of a pair of modulated diode circuits with strong and weak correlations are compared with the results of single re-radiating source measurement, respectively. Coupling path correlation factors are used to quantify the effects of different spatial locations on the

coupling links. It is found that the statistical distributions of the re-radiated spectrum differ when two non-linear sources are present; the differences are dependent on the spatial locations of the sources, the changes being observed when the separation of the sources results in weak correlation of the coupling paths. This is in addition to the changes to the re-emission statistics observed when the chamber input RF power level is changed as reported previously [5, 6]. Further investigations are continuing to extend this work to the general case of multiple non-linearities in order to analyse the effects the multiple spatial locations make on the statistical models of the re-radiated spectrum. Such multiple sources will replicate electronic equipment with realistic complexity.

#### REFERENCES

- [1] I. D. Flintoft, A. C. Marvin, M. P. Robinson, K. Fischer and A. J. Rowell, "The Re-Emission Spectrum of Digital Hardware Subjected to EMI", *IEEE Transactions on Electromagnetic Compatibility*, vol. 45, No. 4, pp. 576-585, Nov. 2003.
- [2] M. P. Robinson, K. Fischer, I. D. Flintoft and A. C. Marvin, "A Simple Model of EMI-Induced Timing Jitter in Digital Circuits, its Statistical Distribution and its Effect on Circuit Performance", *IEEE Transactions on Electromagnetic Compatibility*, vol. 45, no. 3, pp.513-519, Aug. 2003.
- [3] D. A. Hill, "Electronic Mode Stirring for Reverberation Chambers", *IEEE Transactions on Electromagnetic Compatibility*, vol. 36, no. 4, pp. 294-1036, Nov. 1994.
- [4] I. D. Flintoft, A. C. Marvin and L. Dawson, "Statistical Response of Nonlinear Equipment in a Reverberation Chamber", in *2008 IEEE International Symposium on Electromagnetic Compatibility*, Detroit, Aug. 18-22, 2008.
- [5] A. C. Marvin, J. Chen, I. D. Flintoft and J. F. Dawson, "A Describing Function Method for Evaluating the Statistics of the Harmonics Scattered from a Non-Linear Device in a Mode Stirred Chamber", in *2009 IEEE International Symposium on Electromagnetic Compatibility*, Austin, Aug. 17-21, 2009. pp. 165-170.
- [6] J. Chen, A. C. Marvin, I. D. Flintoft and J. F. Dawson, "Double-Weibull Distributions of the Re-Emission Spectra from a Non-Linear Device in a Mode Stirred Chamber", in *2010 IEEE International Symposium on Electromagnetic Compatibility*, Fort Lauderdale, Florida, Jul. 18-23, 2010.
- [7] J. Chen, A. C. Marvin, I. D. Flintoft and J. F. Dawson, "A Statistical Approach to Radiated Immunity Testing of Digital Hardware based in a reverberation chamber", in *EMC Europe 2010, 9th International Symposium on EMC joint with 20th International Wroclaw Symposium on EMC*, Wroclaw, Poland, Sept. 13 -17, 2010.
- [8] C. Lemoine, P. Besnier and M. Drissi, "Investigation of Reverberation Chamber Measurements Through High Power Goodness-of-fit Tests". *IEEE Transactions on Electromagnetic Compatibility*, vol. 49, no. 4, pp. 745-755, Nov. 2007.