Polarisation Vision

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Overview

• Background
• Single View Polarization Vision
• Multiple View Polarization Vision
• Patch Merging
• BRDF Estimation
• Results
• Conclusions
Shape-from-shading

Recover surface normals and hence surface height from observed variations in image brightness,

If surface reflectance is Lambertian, then surface normal lies on a cone whose axis is the light source direction and whose opening angle is the inverse cosine of the normalised image brightness,

Hence zenith angles of surface normals are determined by Lambert’s law, azimuth angles determined by boundary conditions and smoothness constraints.
Geometric SFS

(Worthington and Hancock '99)

Surface normal must fall on a cone whose axis is light source direction and whose opening angle is determined by image brightness.

\[ I = n \cdot s = \cos \theta \]
Polarisation

Method breaks down for surfaces which are non-Lambertian, and this includes those that are shiny (exhibit specularities) and those that are rough.

Aim in this talk is to use the Fresnel theory to show that polarisation measurements can be used to determine the zenith and azimuth angles for shiny surfaces.

We make use of diffuse polarisation. Hence, the incoming light is unpolarised but develops a spontaneous polarisation due to interaction with the surface.

Degree of diffuse polarisation determines the surface normal zenith angle, and the phase-angle the azimuth angle (subject to ambiguity)
Publications relevant to work

• Use diffuse polarisation measurements to estimate surface orientation (IEEE TIP 06).

• Extend to multiple views to resolve ambiguities and extend object coverage (PAMI 07).

• Use method to estimate BRDF’s for surfaces composed of different materials (CVIU 08).

• Explore whether multiple images taken with fixed camera direction and varying light source direction (i.e. photometric stereo) can he used to resolve azimuth angle ambiguity (CAIP 07).
## Polarization Vision and Applications

### Shape from polarization
- **Polarization vision**
  - Wolff and Boult TPAMI ’91
- **Shape recovery**
  - Miyazaki et al ICCV ’03, TPAMI ’04
  - Drbohlav and Šára SPIE ’99
  - Rahmann and Canterakis CVPR ’01

### Other uses of polarization
- **Reflection components**
  - Umeyama TPAMI ’04
- **Photometric stereo**
  - Drbohlav and Šára ICCV ’01
- **Range scanning**
  - Clark, Trucco and Wolff IVC ’97
- **Marine vision**
  - Schechner and Karpel CVPR ’03
- **BRDF estimation**
  - Shibata et al SPIE ’05
- **Segmentation / classification**
  - Chen and Wolff IJCV ’98
An end user

The mantis shrimp
Theoretical background

Fresnel theory
Augustin-Jean Fresnel (1788-1827)
Basic concepts
Theory: Physical Origins of Polarization by Reflection

Specular reflection

Snell’s Law

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

Fresnel Coefficients

Perp to incidence plane

\[ r_\perp \equiv \frac{E_{0r\perp}}{E_{0i\perp}} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \]

\[ R_\perp = r_\perp^2 \]

Parallel to incidence plane

\[ r_\parallel \equiv \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t} \]

\[ R_\parallel = r_\parallel^2 \]
Origins of polarisation

Occurs when light is reflected from boundary between layers of different refractive index
Polarisation for specular reflection

Defined in terms of reflection coefficients for different planes of polarisation.

\[
\rho_s = \frac{R_\perp(n, \theta_i) - R_\parallel(n, \theta_i)}{R_\perp(n, \theta_i) + R_\parallel(n, \theta_i)}
\]

\[
\rho_s = \frac{2 \sin^2 \theta \cos \theta \sqrt{n^2 - \sin^2 \theta}}{n^2 - \sin^2 \theta - n^2 \sin^2 \theta + 2 \sin^2 \theta}
\]
Behaviour

Dielectric $n=1.7$

Metal $n=0.8$, $k=6$. 
Specular polarisation versus incidence angle

Because of Brewster angle. For a measured polarisation there are two possible incidence angles.
Polarisation for diffuse reflection

Defined in terms of transmission rather than reflection: \( T = 1 - R \)

\[
\rho_d = \frac{T_\parallel (1/n, \theta_i') - T_\perp (1/n, \theta_i')}{T_\parallel (1/n, \theta_i') + T_\perp (1/n, \theta_i')} = \frac{R_\perp (1/n, \theta_i') - R_\parallel (1/n, \theta_i')}{2 - R_\perp (1/n, \theta_i') - R_\parallel (1/n, \theta_i')}
\]

Use Snell’s law to re-express in terms of emittance angle

\[
\rho_d = \frac{(n - 1/n^2) \sin^2 \theta}{2 + 2n^2 - (n - 1/n^2) \sin^2 \theta + 4 \cos \theta \sqrt{n^2 - \sin^2 \theta}}
\]
Behaviour

![Graph showing transmission coefficients vs. emittance angle](image-url)
Diffuse polarisation versus emittance angle

No Brewster angle for diffuse polarisation. Single measurement of polarisation gives a single emittance angle.
Theory: Polarization by Reflection

Specular reflection

\[ n_i = 1 \]
\[ n_t = 1.4 \]

Low polarization
Theory: Polarization by Reflection

Specular reflection

High polarization
Theory: Polarization by Reflection

Specular reflection

Complete polarization: Brewster Angle

Reflected light totally extinguished by rotating polariser.
Theory: Polarization by Reflection

Specular reflection

High polarization
Theory: Shape from Diffuse Polarization

Diffuse reflection

Diffuse component emerges after subsurface scattering
Polarisation Camera

1. Acquire polarization images with light source, camera and object fixed while polarizer rotates

Note: incident light is unpolarised.
**Polarisation measurements**

Measured intensity variation

- Rotate polariser and measure brightness at each pixel with camera, light source and object fixed.

- Brightness varies sinusoidally with polariser angle.

- Fit to recover maximum and minimum brightness together with phase of sinusoid at each pixel.

- Compute polarisation from max and min brightnesses.

Degree of polarization:

\[ \rho = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \]
Polarisation Image

- Composed of brightness, phase and polarisation
Single view shape reconstruction

Use estimates of zenith and azimuth angles to recover surface normals. Reconstruct object shape using surface integration.
Single View Shape Recovery: Overview

1. Acquire polarization images

2. Estimate zenith angles from degree of polarization

3. Ambiguously estimate azimuth angles

4. Disambiguate azimuth angles

5. Integrate normals using Frankot-Chellappa method [TPAMI ’88]
Single View Vision: Apparatus

1. Acquire polarization images

Image plane

Polarizer angle

Phase angle

Single light source

Target on turn-table
Single View Vision: Method

A: Estimate zenith angles from degree of polarization

\[ \rho_d = \frac{(n - 1/n)^2 \sin^2 \theta}{2 + 2n^2 - (n + 1/n)^2 \sin^2 \theta + 4 \cos \theta \sqrt{n^2 - \sin^2 \theta}} \]

Single real solution since polarisation increases monotonically with emittance angle. i.e. there is no Brewster angle for diffuse polarisation.
**Single View Vision: Method**

B: Ambiguously estimate azimuth angles from measured phase

\[
\alpha = \begin{cases} 
\phi \\
\phi + \pi 
\end{cases}
\]

Azimuth angle of surface normal is orientation of projection of surface normal onto image plane. Light is reflected most efficiently when polarised parallel to plane containing surface normal and reflected ray. Hence, phase of polarised light is equivalent to azimuth angle of surface normal up to an ambiguity of 180 degrees.
Disambiguation

- On boundary select azimuth angle that is closest to that of occluding boundary normal.

- Propagate constraint as brush-fire into interior of object.

- For small zenith angles allow abrupt changes of azimuth angle.
• Diffuse polarisation solved for surface normal zenith angle (unambiguously)

\[
\cos \theta = \frac{n.s}{L}
\]

• Analogous to shape-from-shading, where Lambert’s law allows zenith angle to be determined from measured image brightness

\[ L = n.s = \cos \theta \]
Single View Vision: Method

Intensity, $I$

Phase, $\phi$

Degree of polarization, $\rho$
Single View Vision: Method

4. Disambiguate azimuth angles
Examples
Results

5. Integrate surface normals
Height functions
Single View Vision: Accuracy
Single View Vision: Limitations

- Ambiguities
- Roughness modifies dependance on zenith angle.
- At specularities use wrong $\rho(\theta)$
- Inter-reflections
Single View Vision: Refractive Index

- Linear polarizer
  - Placed horizontally
- Rotating table
- Sample
- Screen
- Laser
Resolving some of the problems: PAMI07

- Make radiance function estimation to recover azimuth angle more accurately (implemented as look-up table).

- Use multi-view correspondences to resolve ambiguities in surface normal direction.

- Increase object coverage using multiple views.
Schematic for multi-view approach

Use polarization to estimate the zenith angles from (5) and the two possible azimuth angles from the polarization angle.

Refine zenith angle estimates using the intensity and the reflectance function.

Correspondence to be established between resulting patches.

View 1 estimates

View 2 estimates

Reflectance function estimation

Enhanced data

Enhanced data

Segment into patches

Segment into patches

Calculate alignment parameters and matching costs for patch pairs using surface normals and topography

Acquire correspondence from patch pairs with lowest cost

Use interpolation to estimate remaining correspondences

Integrate unambiguous surface normals to recover depth

Depth

[§3.3]

[§4.1]

[§4.2-3]

[§4.3]

[§4.4]

[Frankot and Chellappa 1988]
BRDF Estimation

Use estimates of zenith and azimuth angles to explore angular dependence of reflected surface radiance.
Polarisation Image

- Light through a polaroid: \( I(\alpha_p) = I_o [1 + \rho \cos(2\alpha_p - 2\phi)] \) \hspace{1cm} \text{...(1)}

- Degree of diffuse polarisation

\[
\rho = \frac{(n - 1/n)^2 \sin^2 \theta}{2 + 2n^2 - (n - 1/n)^2 \sin^2 \theta + 4 \cos \theta \sqrt{n^2 \sin^2 \theta}}
\] \hspace{1cm} \text{...(2)}

- From the Fresnel theory, azimuth angle equals \( \phi \) in (1); zenith angle can be computed from (2) given \( \phi \) and \( n \).
Reflectance Distributions

**Natural leaves**

- Normalized pixel intensity vs. Degree of polarization
- Normalized pixel intensity vs. Polarization phase

**Plastic leaves**

- Normalized pixel intensity vs. Degree of polarization
- Normalized pixel intensity vs. Polarization phase
Feature Generation

- Spherical Harmonic coefficient definition
  \[ a_{l,m} = \int_{0}^{2\pi} \int_{0}^{\pi} f(\theta,\phi) Y_{l}^{m}(\theta,\phi) \sin \theta \, d\theta \, d\phi \]  
  \( \ldots (3) \)

- Moment estimates of coefficients
  \[ a_{l,m} = \frac{1}{M} \sum_{i=1}^{M} \hat{I}_{i} Y_{l}^{m}(\theta_{i},\phi_{i}) \]  
  \( \ldots (4) \)

- Mahalanobis distance between the feature vectors for segmentation by normalized graph cuts.

*PCA mapped features - 4 dimensions with greatest variance*
Experimental Results

• Segmentation Results:

  Test Image  |  Natural surface  |  Artificial surface
  ![Test Image](image1.png) | ![Natural surface](image2.png) | ![Artificial surface](image3.png)
Photometric stereo

Fixed camera and object, variable light source direction.
• However, there is an ambiguity of 180 degrees in azimuth angle determination.

• Resolve ambiguity using light source consistency constraints.
• Photometric stereo setup: polarization camera and object kept fixed but light source is moved.
Resolving angle ambiguities

- Azimuth angles are disambiguated differently depending on whether the phase angle is less than 45° (regions A and A’), between 45° and 135° (regions B and B’) or greater than 135° (regions C and C’).
Light source consistency constraints

- Resolve azimuth angle ambiguities using light source consistency constraints (for spherical surface):

\[
\text{if } \phi_k < 45^\circ \text{ then } \alpha_k = \begin{cases} 
\phi_k & \text{if } I_k^{(2)} > I_k^{(1)} \\
\phi_k + 180^\circ & \text{otherwise}
\end{cases}
\]

\[
\text{if } 45^\circ \leq \phi_k < 135^\circ \text{ then } \alpha_k = \begin{cases} 
\phi_k & \text{if } I_k^{(3)} > I_k^{(1)} \\
\phi_k + 180^\circ & \text{otherwise}
\end{cases}
\]

\[
\text{if } 135^\circ \leq \phi_k \text{ then } \alpha_k = \begin{cases} 
\phi_k & \text{if } I_k^{(3)} > I_k^{(2)} \\
\phi_k + 180^\circ & \text{otherwise}
\end{cases}
\]
Examples of disambiguation

• Raw images (top) and disambiguated azimuthal angles (bottom):
Results

- Reconstructed surfaces
Accuracy

• Comparing with ground truth range data:

Profile of the vase reconstruction using

(a) raw zenith angle estimates,

(b) zenith angles estimated using the Miyazaki et al. method,

(c) zenith angles using method reported here,

(d) exact profile is shown by the thick line.
Shape and Refractive Index from Spectro-polarimetry
Idea

• Multiple polarisation images from a single viewpoint and different wavelengths

• Additional constraints on a) wavelength and b) surface integrability.

• Solved using optimisation method,
Physics

From the Fresnel equations

\[
\frac{I_{\text{min}}}{I_{\text{max}}} = \left( \frac{\cos \theta(u) \sqrt{\eta^2(u, \lambda) - \sin^2 \theta(u) + \sin^2 \theta(u)}}{\eta(u, \lambda)} \right)^2 = R(u, \lambda)^2
\]

Solve for zenith angle

\[
\sin \theta(u) \equiv \frac{\eta(u, \lambda) \sqrt{1 - R^2(u, \lambda)}}{\sqrt{\eta^2(u, \lambda) - 2R(u, \lambda)\eta(u, \lambda) + 1}}.
\]
Material Dispersion Equations

Need model of wavelength dependence of refractive index

Cauchy

\[ \eta(u, \lambda) = \sum_{m=1}^{M} C_m(u) \lambda^{-2(m-1)}, \]

Sellmeier

\[ \eta^2(u, \lambda) = 1 + \sum_{m=1}^{M} \frac{B_m(u) \lambda^2}{\lambda^2 - D_m(u)}, \]
Integrability Constraint

Constraints on second mixed derivatives of height function can be reexpressed in terms of variation in zenith and azimuth angles.

\[
\cos \alpha(u) \frac{\partial \tan \theta(u)}{\partial y} = \sin \alpha(u) \frac{\partial \tan \theta(u)}{\partial x}.
\]

In terms of rate of change of zenith angle with position on pixel lattice

\[
\cos \alpha(u) \theta_y(u) = \sin \alpha(u) \theta_x(u)
\]
Cost Function

Sum of terms from data-closeness of Fresnel transmission ratio and integrability constraint over image pixels and wavelengths

\[ E = \int_S \int_{\mathcal{W}} \left( R(u, \lambda) - r(u, \lambda) \right)^2 d\lambda \, du + \beta(u) \int_S \left( \cos \alpha(u) \theta_\gamma(u) - \sin \alpha(u) \theta_\times(u) \right)^2 du, \]
Method

• Collect images with fixed viewpoints at different light source, polariser and wavelength settings.

• Solve minimisation problems for refractive index and zenith angle

• Use wavelength dependant phase information for resolve azimuth ambiguity.
Results
Depth maps
Refractive Index Variation
Wavelength Dependence of Refractive Index
Conclusions

• Demonstrated potential of diffuse polarisation for shape-recovery from single and multiple polarisation images.

• Gives reliable shape recovery, and could be the basis of a range imaging camera design.

• Currently exploring possibilities of using method for fruit quality control.